

Name: _____

Instructor: _____

**Math 10550, Exam I
September 18, 2012**

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- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 9 pages of the test.

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Multiple Choice _____

11. _____

12. _____

13. _____

Total _____

Name: _____

Instructor: _____

Multiple Choice

1.(6 pts.) The function $f(x) = \frac{x^2 - 1}{x^3 - 4x}$ is continuous everywhere *except* at

- (a) $x = 0$ and $x = \pm 2$
- (b) $x = 0, x = \pm 1$ and $x = \pm 2$
- (c) $x = 0$ and $x = \pm 1$
- (d) $x = \pm 2$
- (e) f is a rational function and so it is continuous everywhere

2.(6 pts.) Compute

$$\lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 5} - 3}{x - 2}.$$

- (a) $\frac{1}{2}$
- (b) 0
- (c) $\frac{2}{3}$
- (d) $\frac{1}{3}$
- (e) 1

Name: _____

Instructor: _____

3.(6 pts.) Find $f'(4)$ if

$$f(x) = 4\sqrt{x} - \frac{16}{\sqrt{x}}.$$

- (a) 4 (b) 3 (c) 0 (d) -1 (e) 2

4.(6 pts.) Compute the derivative of

$$f(x) = \frac{x + \cos x}{x + \sin x}.$$

- (a) $\frac{1 - \sin x}{1 + \cos x}$
- (b) $\frac{(1 - \sin x)(x + \sin x) - (x + \cos x)(1 + \cos x)}{(x + \sin x)^2}$
- (c) $\frac{(1 - \sin x)(x + \sin x) + (x + \cos x)(1 + \cos x)}{(x + \sin x)^2}$
- (d) $\frac{(1 - \sin x)(x + \sin x) - (x + \cos x)(1 + \cos x)}{(1 + \cos x)^2}$
- (e) $\frac{(1 - \cos x)(x + \sin x) - (x + \cos x)(1 + \sin x)}{(x + \sin x)^2}$

Name: _____

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5.(6 pts.) All the horizontal tangent lines to the curve $y = \frac{1}{1+x^2}$ are given by

$$(a) \quad y = \frac{1}{2}$$

$$(b) \quad y = \frac{1}{3}$$

(c) $y \equiv 1$

$$(d) \quad y = \frac{1 - x^2}{(1 + x^2)^2}$$

$$(e) \quad y = \pm \frac{1}{2}$$

6.(6 pts.) Find the derivative of $f(x) = (1 + \sin(x^2))^{1/4}$.

$$(a) \quad \frac{1}{2}x(1 + \sin(x^2))^{-3/4} \sin(x^2)$$

$$(b) \quad \frac{1}{4}(1 + \sin(x^2))^{-3/4}$$

$$(c) \quad \frac{1}{4}(1 + \sin(x^2))^{-3/4} \cos(x^2)$$

$$(d) \quad \frac{1}{2}x(1 + \sin(x^2))^{-3/4} \cos(x^2)$$

$$(e) \quad \frac{1}{4}(1 + \sin(x^2))^{-3/4} \sin(x^2)$$

Name: _____

Instructor: _____

7.(6 pts.) If $f(x) = x^2 \cos x + \sin x$, find $f''(x)$.

- (a) $f''(x) = 2 \cos x - 4x \sin x + x^2 \cos x - \sin x$
- (b) $f''(x) = 4 \cos x + 4x \sin x - 2x^2 \cos x - \sin x$
- (c) $f''(x) = 4 \cos x - 4x \sin x + x^2 \cos x - \sin x$
- (d) $f''(x) = 2 \cos x + 2x \sin x - x^2 \cos x - \sin x$
- (e) $f''(x) = 2 \cos x - 4x \sin x - x^2 \cos x - \sin x$

8.(6 pts.) Compute

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x \tan x}.$$

Hint: Multiply by $\frac{1 + \cos x}{1 + \cos x}$.

- (a) 2
- (b) $\frac{1}{2}$
- (c) 1
- (d) Does not exist.
- (e) 0

Name: _____

Instructor: _____

9.(6 pts.) If $f'(2) = 5$, $g(4) = 2$, $g(2) = 1$, $f(2) = -1$ and $g'(4) = 3$, find $(f \circ g)'(4)$.

- (a) -30 (b) 6 (c) 15 (d) -15 (e) 30

10.(6 pts.) If $\sin(\pi xy) = \pi(x + y)$ find $\frac{dy}{dx}$ at $(1, -1)$ by implicit differentiation.

- (a) 1 (b) $\frac{\pi}{2}$ (c) π (d) 0 (e) -1

Name: _____

Instructor: _____

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(13 pts.) Find the derivative of

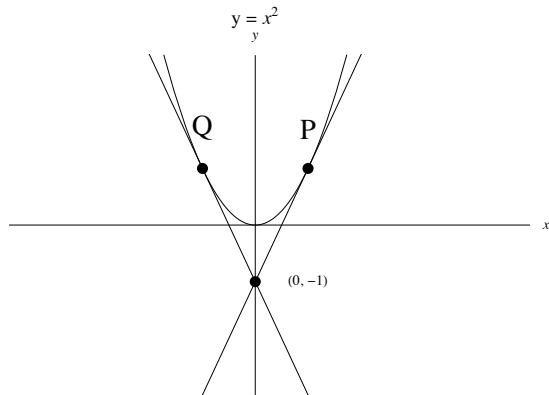
$$y = \frac{1}{1-x}$$

using the definition of the derivative.

Name: _____

Instructor: _____

12.(14 pts.) For what points, P and Q on the graph of the function $y = x^2$ does the tangent line at that point pass through the point $(0, -1)$?



Hint: Write down the equation for the tangent line through the point (a, a^2) and proceed from there.

P : _____

Q : _____

Name: _____

Instructor: _____

13.(13 pts.) Show that there is at least one solution of the equation

$$x^3 = 3x^2 - 1.$$

Justify your answer, identify the theorem you use and explain why the theorem applies.

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